PART : III MATHEMATICS

MCQ type

SECTION – 1 : (Maximum Marks : 24)

- This section contains **SIX (06)** questions.
- Each question has **FOUR options** for correct answer(s). **ONE OR MORE THAN ONE** of these four option(s) is (are) correct option(s).
- For each question, choose the correct option(s) to answer the question.
- Answer to each question will be evaluated according to the following marking chosen.

Full Marks	:	**+4** If only (all) the correct option(s) is (are) chosen.
Partial Marks	:	**+3** If all the four options are correct but ONLY three options are chosen.
Partial Marks	:	**+2** If three or more options are correct but but ONLY two options are chosen, both of which are correct options.
Partial Marks	:	**+1** If two or more options are correct but ONLY one option is chosen and it is a correct option.
Zero Marks	:	0 If none of the options is chosen (i.e. the question is unanswered).
Negative Marks	:	**–2** In all other cases.

For Example : If first, third and fourth are the ONLY three correct options for a question with second option being an incorrect option; selecting only two of the three correct options (e.g. the first and fourth options), without selecting any incorrect option (second option in this case), will result in +2 marks. Selecting only one of the three correct options (either first or third or fourth option), without selecting any incorrect option (second option in this case), will result in +1 marks. Selecting any incorrect option(s) (second option in this case), with or without selection of any correct option(s) will result in –2 marks.

1. In a ΔABC, $a = 10$, $b = 10\sqrt{3}$, $\angle C = 30°$ then which of the following is/are correct ?

(A) Area of circumcircle = 100 m

(B) Radius of incircle = $10\sqrt{3} - 15$

(C) $\angle B = 45°$

(D) $\angle A = 120°$

Ans. **(AB)**

Sol. $\cos C = \dfrac{a^2+b^2-c^2}{2ab}$

$\dfrac{\sqrt{3}}{7} = \dfrac{100+300-c^2}{2.10.10\sqrt{3}}$

$300 = 400 - c^2$

$c^2 = 100$

$c = 10$

$\Delta = \dfrac{1}{2}ab\sin C = \dfrac{1}{2}10.10\sqrt{3} \times \dfrac{1}{2} = 25\sqrt{3}$

$r = \dfrac{\Delta}{s} = \dfrac{25\sqrt{3}\times 2}{(20+10\sqrt{3})} = \dfrac{50\sqrt{3}}{20+10\sqrt{3}} = \dfrac{5\sqrt{3}}{2+\sqrt{3}} \times \dfrac{2-\sqrt{3}}{2-\sqrt{3}} = 5(2\sqrt{3}-3) = 10\sqrt{3}-15$

by sine rule $\dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

$\dfrac{10\sqrt{3}}{\sin B} = \dfrac{10}{\sin C} \Rightarrow C = 30$

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$2R = \dfrac{c}{} = \dfrac{10}{} \Rightarrow R = 10$

$\sin C = 1/2$

Hence area of circumcircle = $\pi R^2 = 100\pi$

2. If $f'(x) = e^{(f(x)-g(x))}\, g'(x)$ and $f(1) = g(2) = 1$

(A) $f(2) > 1 - \ell n2$ (B) $f(2) < 1 - \ell n2$

(C) $g(1) > 1 - \ell n2$ (D) $g(1) < 1 - \ell n 2$

Ans. **(BC)**

Sol. $f'(x) = e^{f(x)-g(x)}\, g'(x) : f(1) = g(2) = 1$

$e^{-f(x)} = e^{-g(x)} + c$

$e^{-f(x)} \cdot f'(x) = e^{-g(x)} \cdot g'(x)$

$\int d(e^{-f(x)}) = \int d(e^{-g(x)})$

$e^{-f(x)} = e^{-g(x)} + c$

$x = 1 \quad \frac{1}{e} = e^{-g(1)} + c$

$x = 2 \quad e^{-f(2)} = \frac{1}{e} + c$

$\therefore \quad g(1) > 1 - \ell n2$

$e^{-f(2)} = 2e^{-1} - e^{-g(1)}$

$e^{-f(2)} = 2e^{-1} - e^{-g(1)}$

$f(2) < 1 - \ell n2$

$e^{-1} - e^{-f(2)} = e^{-g(1)} - e^{-1} \Rightarrow e^{-g(1)} + e^{-f(2)} = 2e^{-1}$

$e^{-g(1)} < 2e^{-1}$

$-g(1) < \ell n2 - 1$

3. If s, r, t be non zero complex numbers and $z = x + iy$ where $sz + t\bar{z} + r = 0$ where $\bar{z} = x - iy$

(A) For z to have unique solution $|s| \neq |t|$

(B) (

(C) If $|s| = |t|$ then z will have z will have infinite solution

(D) If z has more than one solution then it will have infinite solution

Ans. **(AD)**

Sol. $sz + t\bar{z} + r = 0, \ \bar{z} = x - iy$

$\bar{s}\bar{z} + \bar{t}z + r = 0$

(1) + (2)

$(t + \bar{s})\bar{z} + (s + \bar{t})z + (r + \bar{r}) = 0$

$(\bar{t} - \bar{s})\bar{z} + (s - \bar{t})z + (\bar{r} - \bar{r}) = 0$

For unique solution

$\frac{t + \bar{s}}{\bar{t} - \bar{s}} \neq \frac{s + \bar{t}}{s - \bar{t}}$

On solving the above equation we get

$|t| \neq |s|$

$\therefore$ option (A) is correct

Lines overlap if

$\frac{t + \bar{s}}{\bar{t} - \bar{s}} = \frac{\bar{t} + s}{s - \bar{t}} = \frac{r + \bar{r}}{r - \bar{r}}$

$|t| = |s|$ $\quad tr - t\bar{r} + sr - s\bar{r} = sr + s\bar{r} - tr - t\bar{r}$

$2\bar{t}r = 2s\bar{r}$

$\bar{t}r = s\bar{r}$

$\therefore |\bar{t}||r| = |s||r|$

$\therefore |t| = |s|$

$\therefore$ If $|t| = |s|$, lines will be parallel for sure but it may not be coincident

Hence option (c) is wrong & (D) is correct.

4. Let $f\left(\frac{\pi}{6}\right) = -\frac{\pi}{12}$ and $\lim\limits_{t\to x} \frac{f(t)\sin x - \sin t\, f(x)}{t-x} = \sin^2 x$

(A) $f\left(\frac{\pi}{4}\right) = \frac{\pi}{4\sqrt{2}}$ (B) $f''\left(\frac{\pi}{2}\right) + f\left(\frac{\pi}{2}\right) = 0$

(C) $f(x) < \frac{x^4}{6} - x^2$ for $x \in (0, \pi)$ (D) $f'(\alpha) = 0$ for at least one $\alpha \in (0, \pi)$

Ans. **(ABCD)**

Sol. $\lim\limits_{t\to x} \frac{f(x)\sin t - f(t)\sin x}{t-x} = \sin^2 x$

$\frac{f(x)\cos x - f'(x)\sin x}{\sin^2 x} = 1$

$-d\left(\frac{f(x)}{\sin x}\right) = 1$

$\frac{f(x)}{\sin x} = x + c \quad \because \quad f\left(\frac{\pi}{6}\right) = -\frac{\pi}{12} \Rightarrow \quad c = 0 \Rightarrow \quad f(x) = -x\sin x$

(A) $f(x) + f''(x) = -2\cos x$

$f\left(\frac{\pi}{2}\right) + f''\left(\frac{\pi}{2}\right) = 0$

(B) $f\left(\frac{\pi}{4}\right) = \left(\frac{\pi}{4\sqrt{2}}\right)$

(C) $f(x)$ is continuous and differentiable and $f(0) = f(x) = 0$

Using LMVT $f'(c) = 0$ for some $x \in (0, \pi)$

(D) $g(x) = \_\, x\sin x + x^2 \_\, \frac{x^4}{6}$

$g'(x) = f'(x) + 2x - \frac{2x^3}{3}$